On the hardness of RL with value-function approximation

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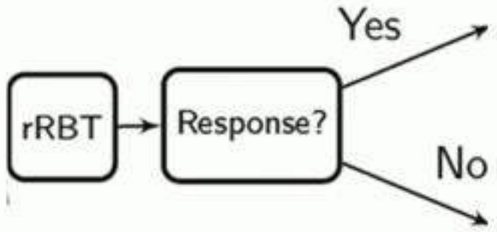
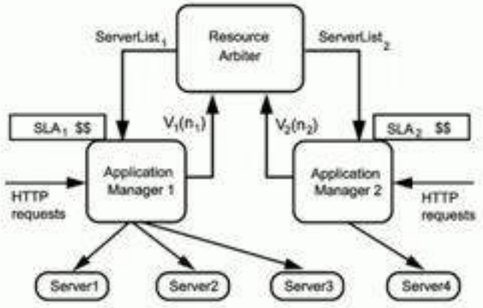
@ MSR Redmond

Based on: *Information-Theoretic Considerations in Batch Reinforcement Learning.*
to appear at ICML-19, with Jinglin Chen (UIUC)

Reinforcement Learning (RL) Applications





[Levine et al'16] [Ng et al'03] [Singh et al'02] [Lei et al'12]
[Mandel et al'16] [Tesauro et al'07] [Mnih et al'15] [Silver et al'16]

Overview

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  - Provides understanding to popular deep RL algorithms, e.g., DQN
- When can we guarantee sample-efficient learning?

Comparison between SL and RL

- Supervised learning
  - Data: $(x, y) \sim P_{X,Y}$
  - A class of predictors F (assume finite), one of which is good
  - Can find a good predictor w/ $O(\log|F|)$ samples (info-theoretic)

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We don't know
… but we should.

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Video game playing

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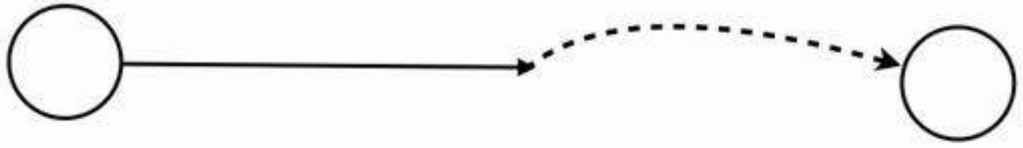

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reward $r_h = R(s_h, a_h)$

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+20

next state $s_{h+1} \in S$

action $a_h \in A$

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e.g., random spawn of enemies

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transition dynamics $P(\cdot \mid s_h, a_h)$ (unknown)

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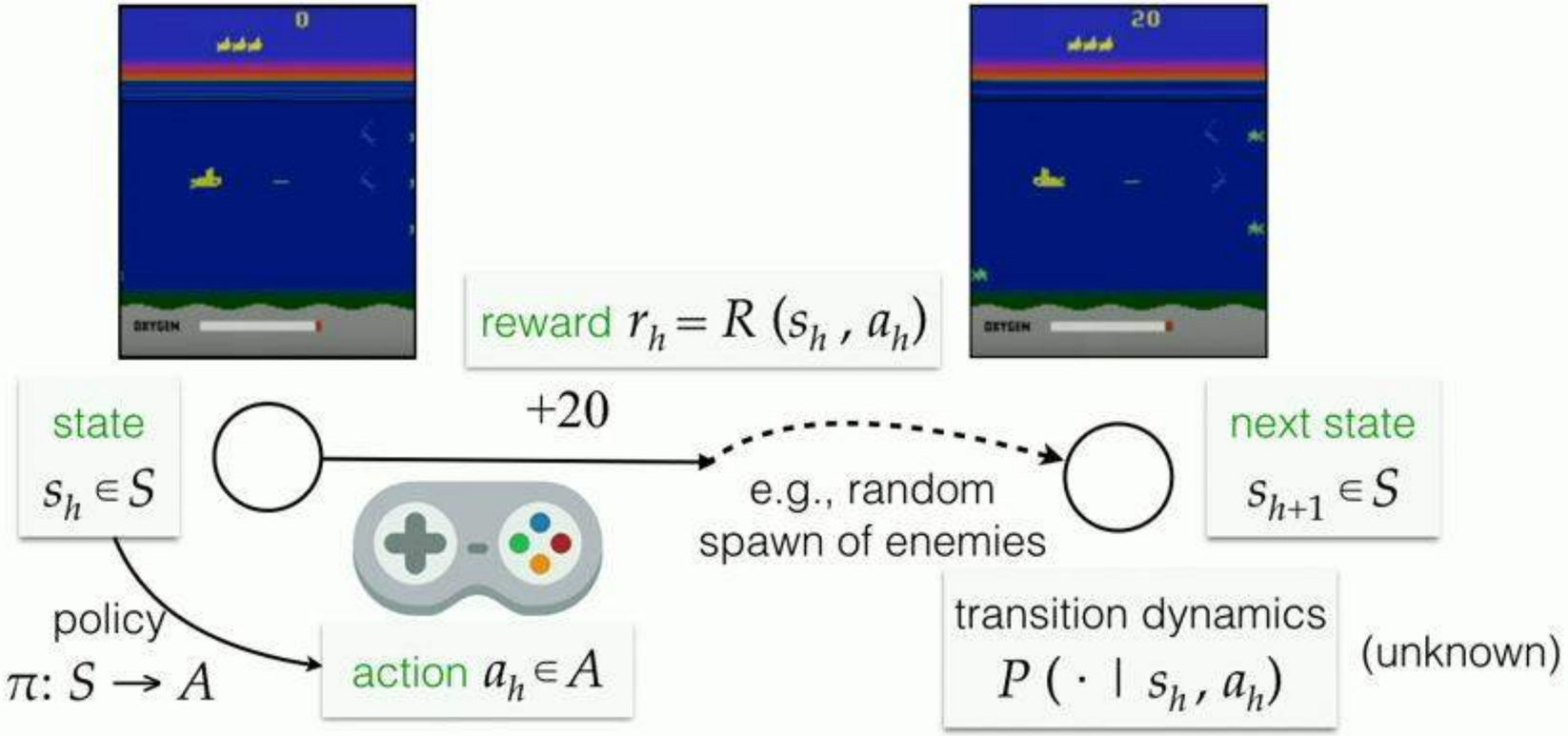

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  - Bellman eq: $Q^\star(s,a) = R(s,a) + \gamma \mathbb{E}_{s' \sim P(s,a)}[\max_{a'} Q^\star(s', a')]$
  - Optimal policy π^* is greedy w.r.t. Q^*

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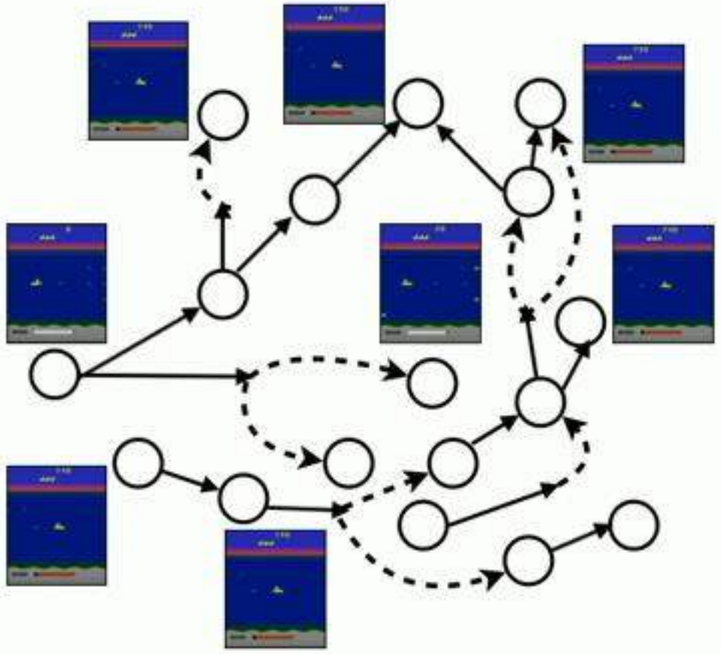

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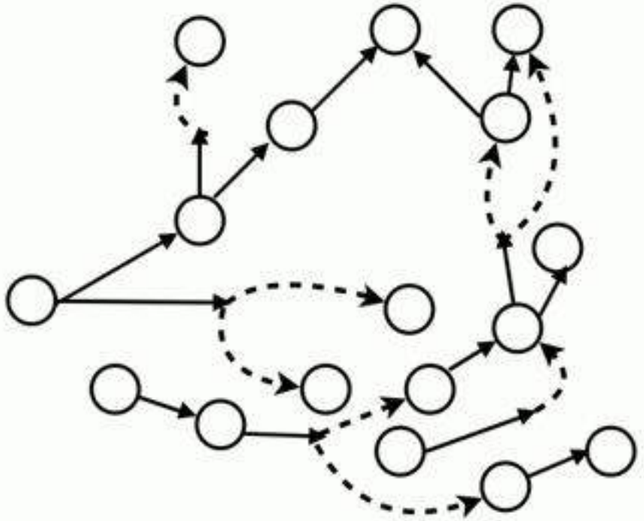

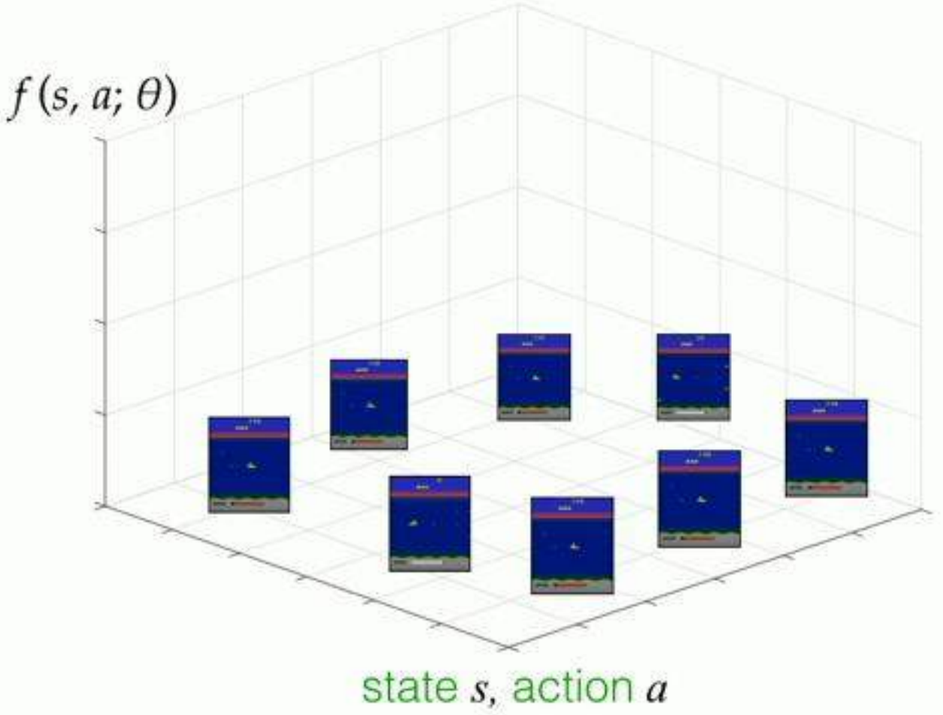

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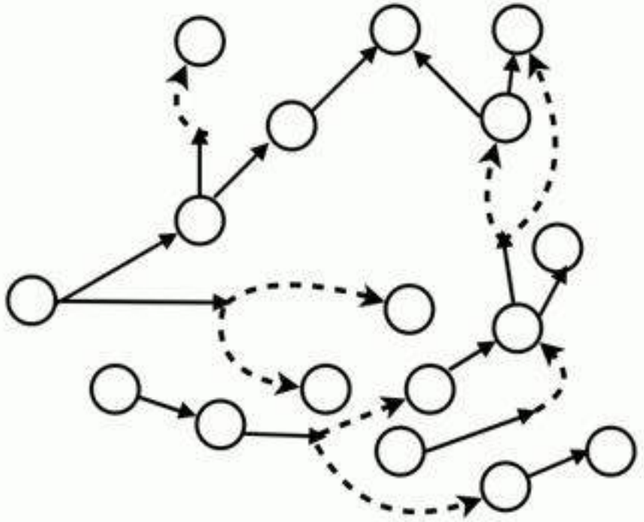

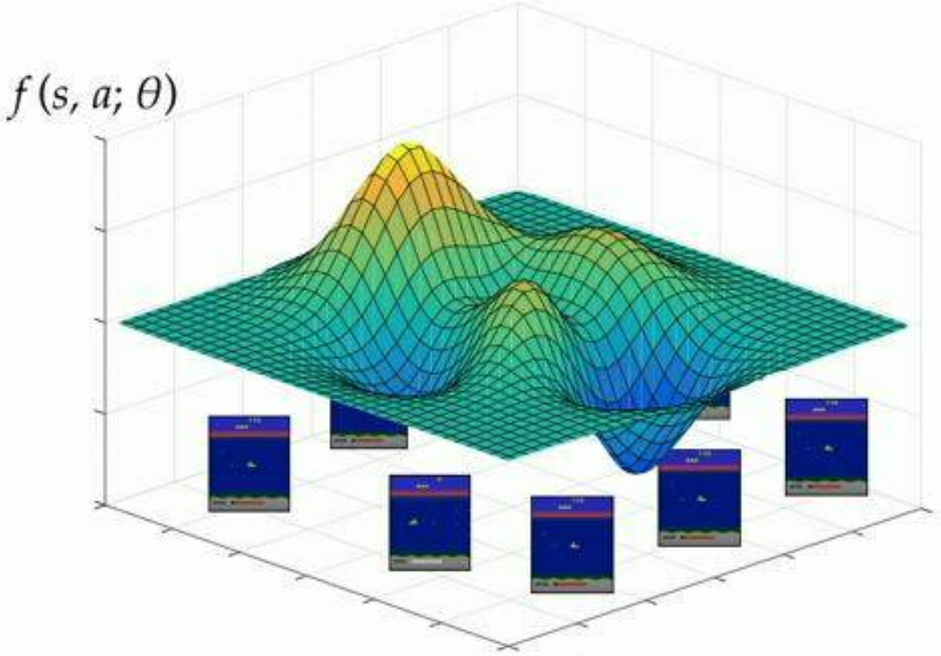

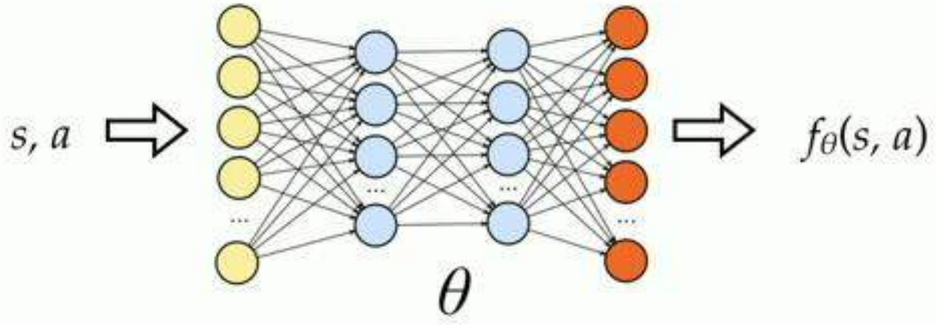

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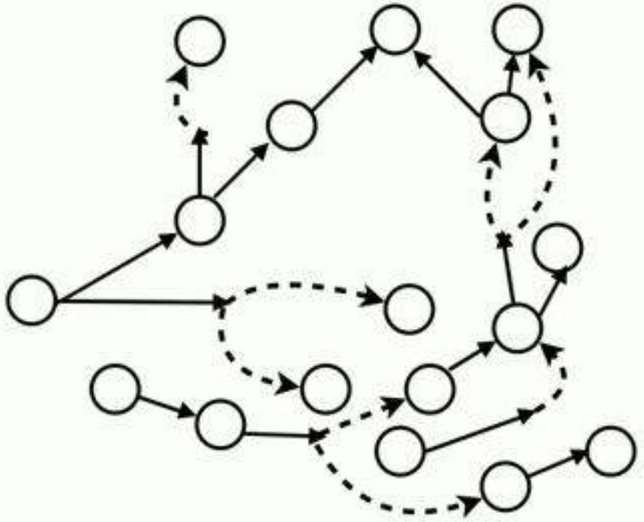

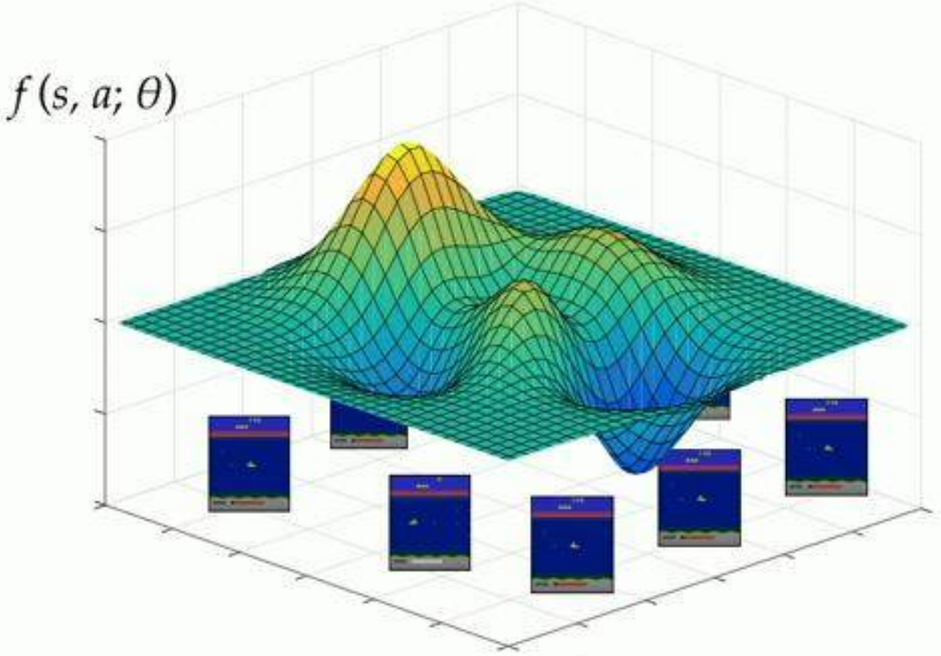

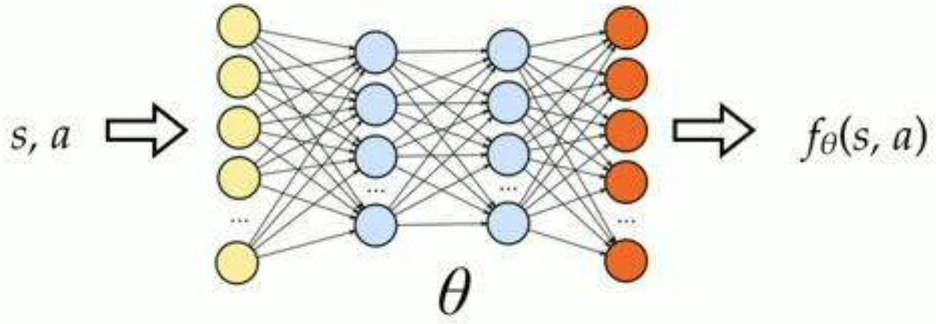

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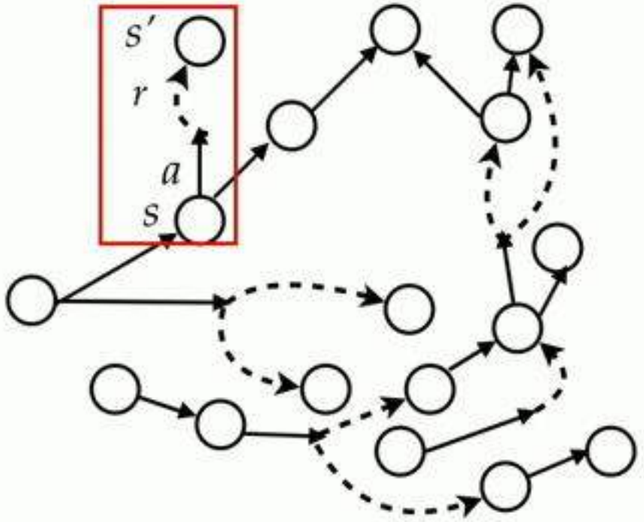

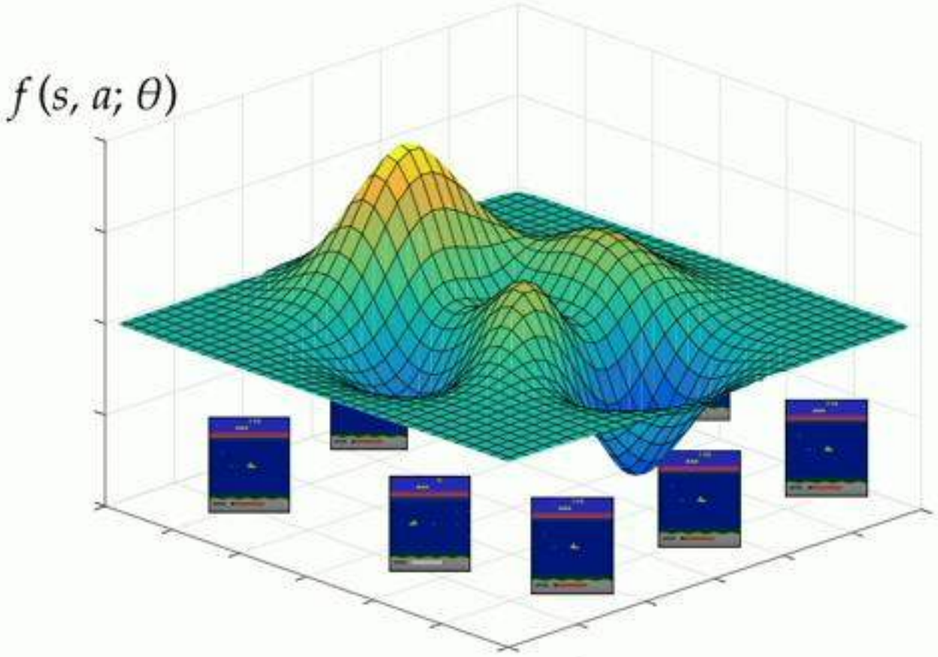

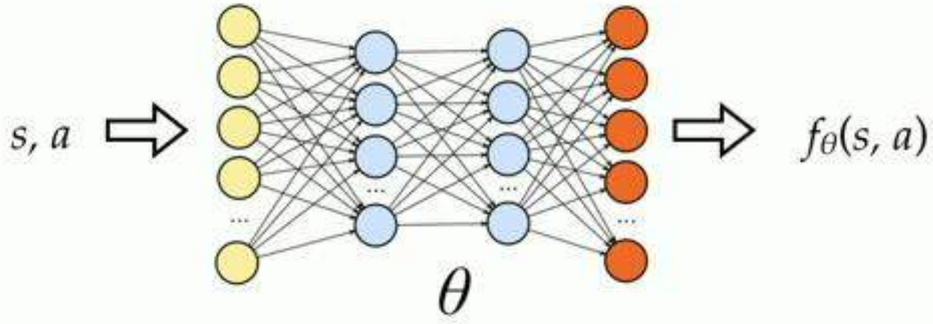

Find θ s.t. $f_\theta(s, a) \approx \mathrm{E}[\, r + \gamma \max_{a'} f_\theta(s', a')\,] \quad \Rightarrow \quad f_\theta \approx Q^*$

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Batch learning in large MDPs

- Dataset $D = \{(s, a, r, s')\}$
  - $(s, a) \sim \mu$ ("data distribution"), $r \sim R(s, a)$, $s' \sim P(s, a)$
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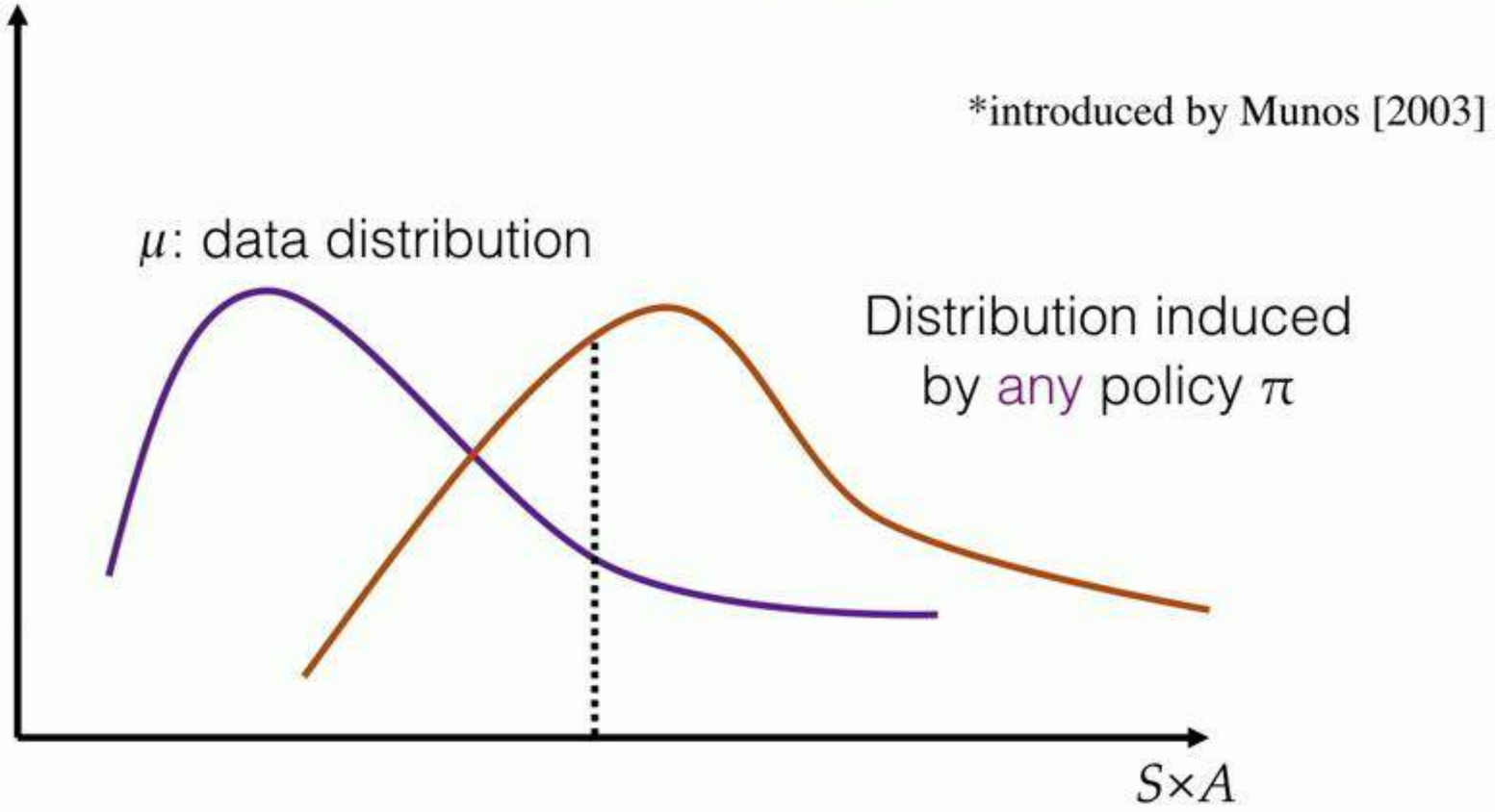
- On function class: F is closed under Bellman update

Assumption on data

*introduced by Munos [2003]



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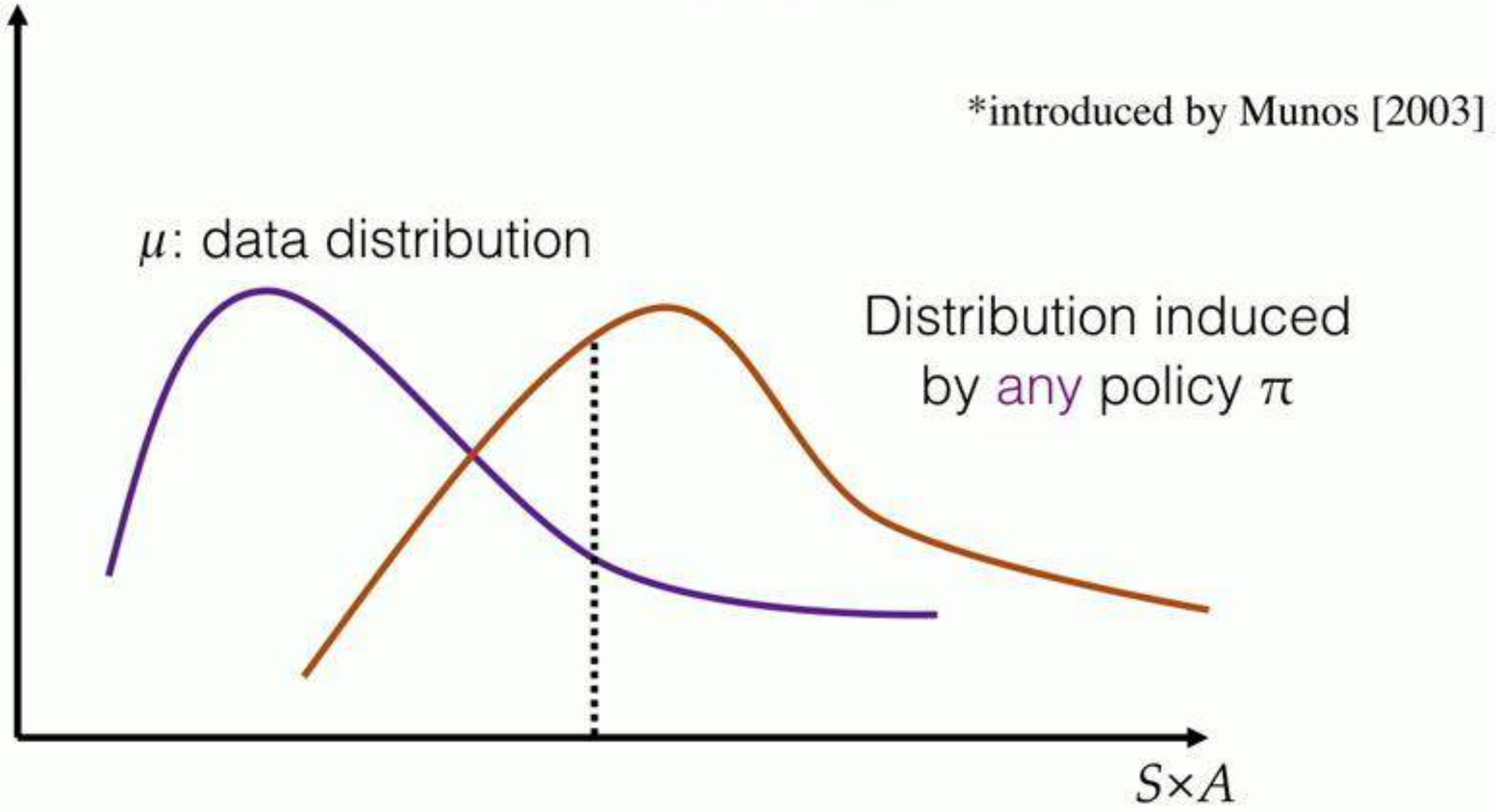

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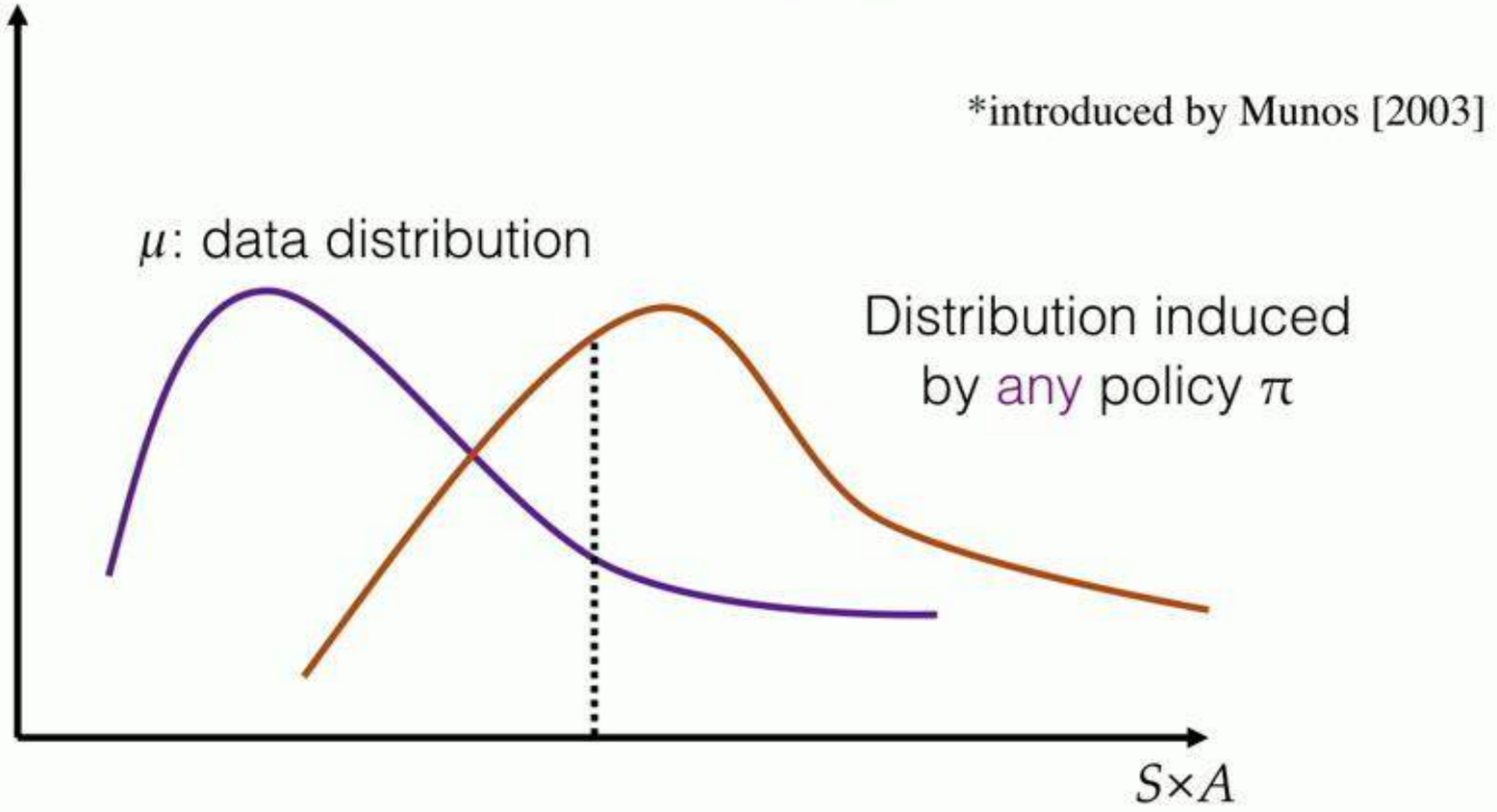

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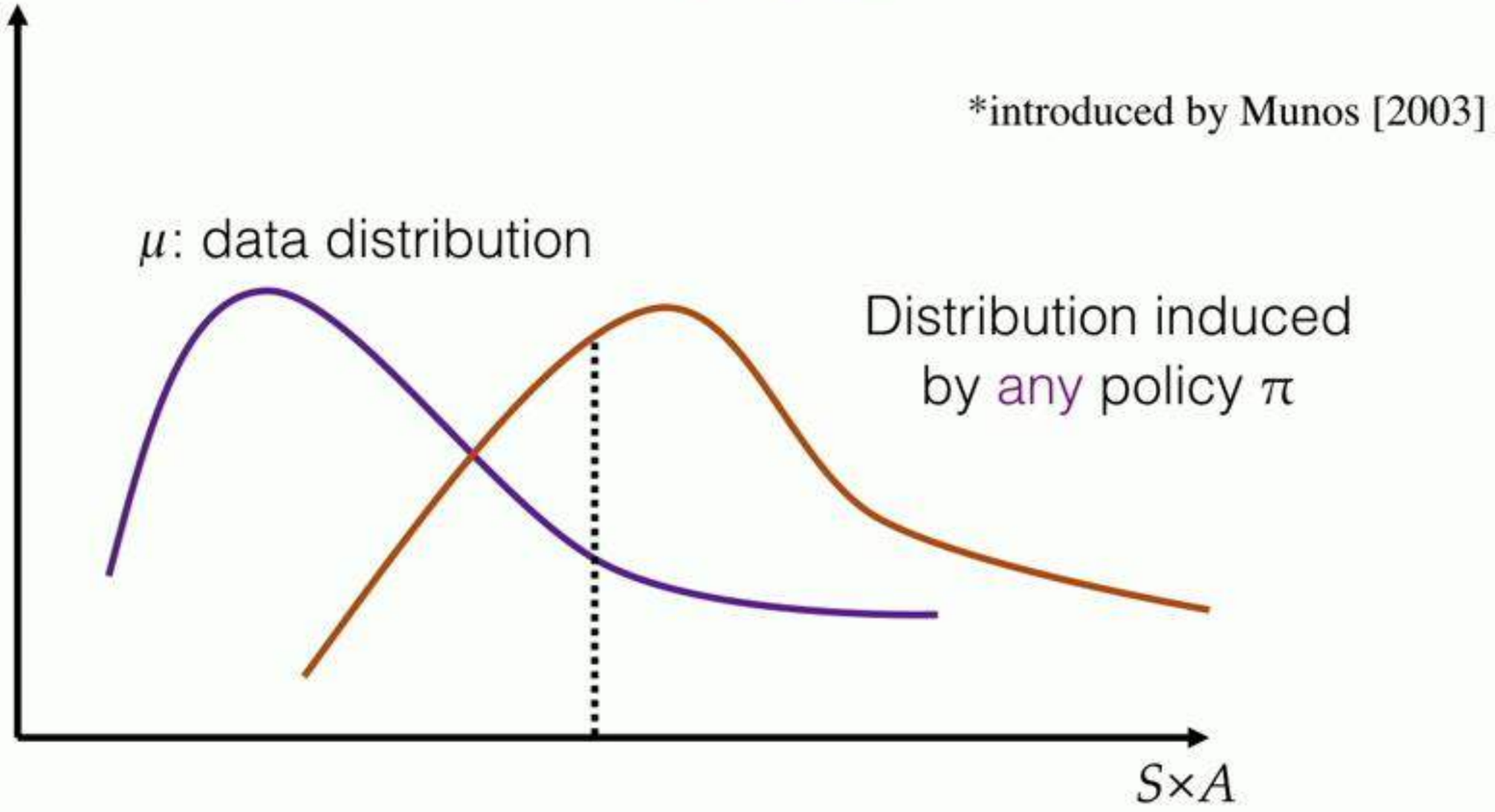

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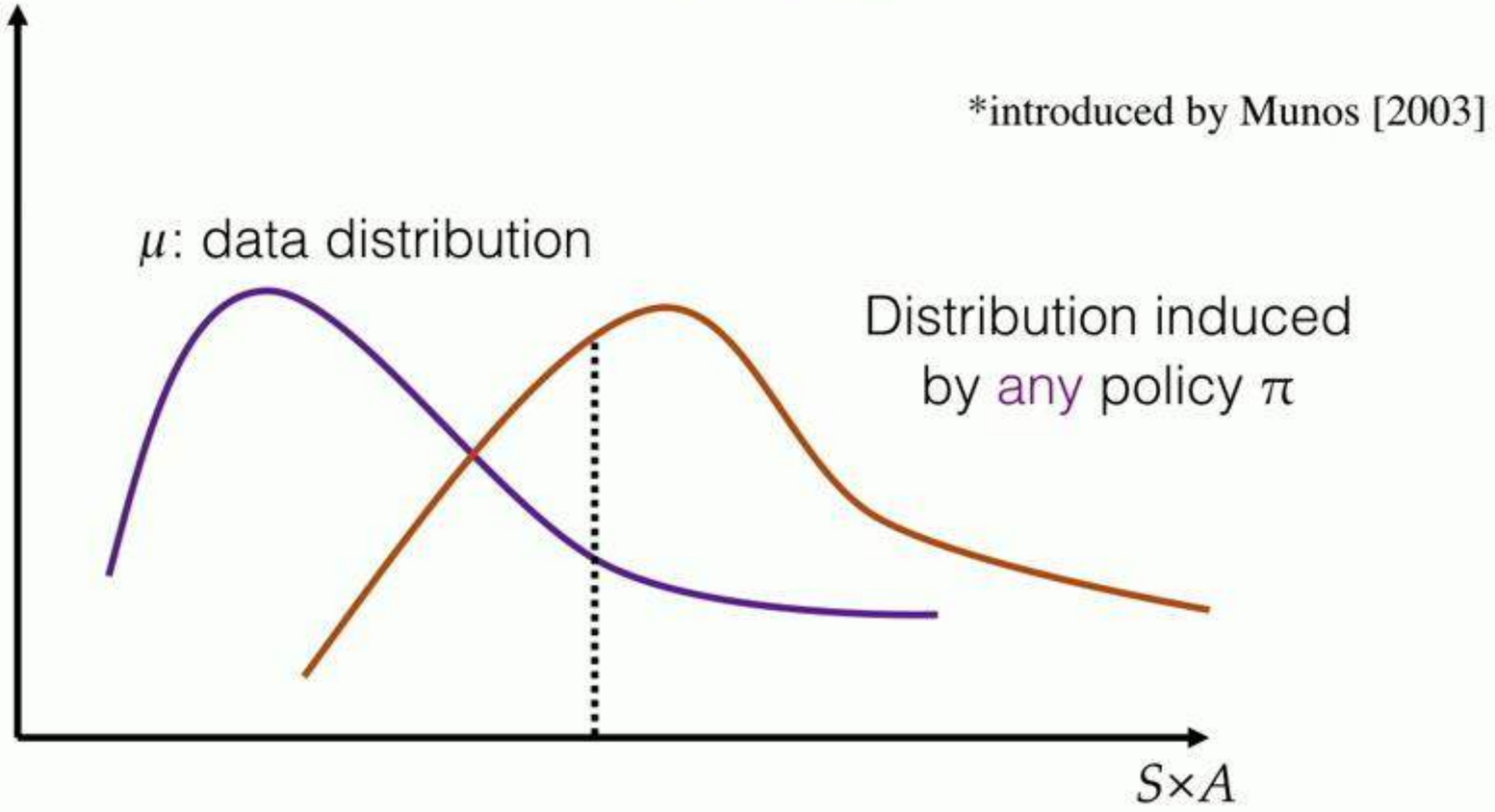

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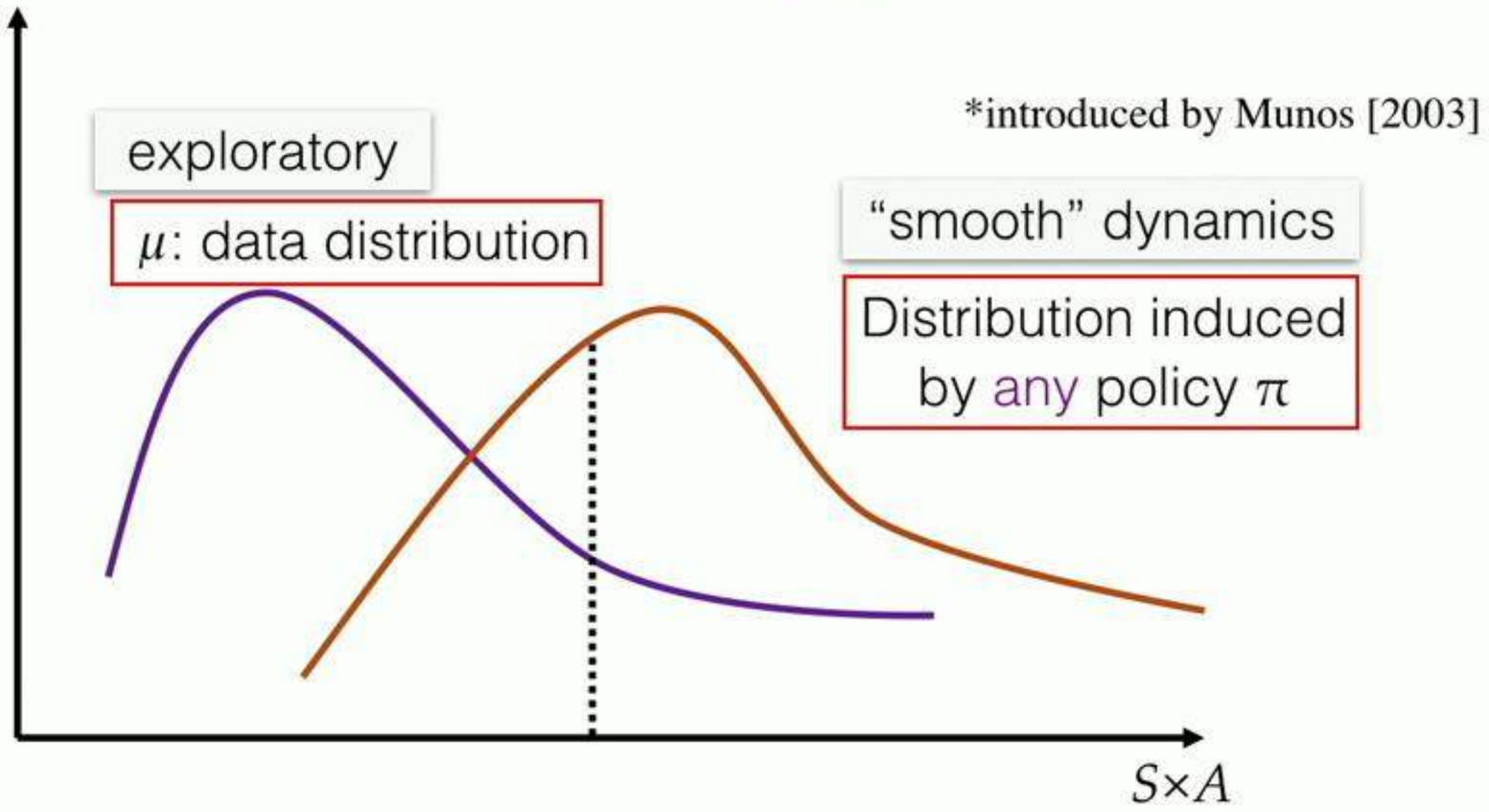

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holds under the most exploratory data distribution!

Batch learning in large MDPs

- Dataset $D = \{(s, a, r, s')\}$
  - $(s, a) \sim \mu$ ("data distribution"), $r \sim R(s, a)$, $s' \sim P(s, a)$
- Function class F s.t. $Q^* \in F$ (finite)
- Goal: find $f \approx Q^*$ s.t. its greedy policy is ε-optimal
  - Recall $v^\pi := \mathbb{E}[\sum_{h=1}^{\infty} \gamma^{h-1} r_h | s_1 \sim \nu_0; \pi]$

Back to the earlier question:

Can we learn such f with a sample complexity poly($\log|F|, H, 1/\varepsilon, 1/\delta$)?

Yes—with two more assumptions

- On data: No policy induces distribution "too different" from μ
- On function class: e.g., F is closed under Bellman update

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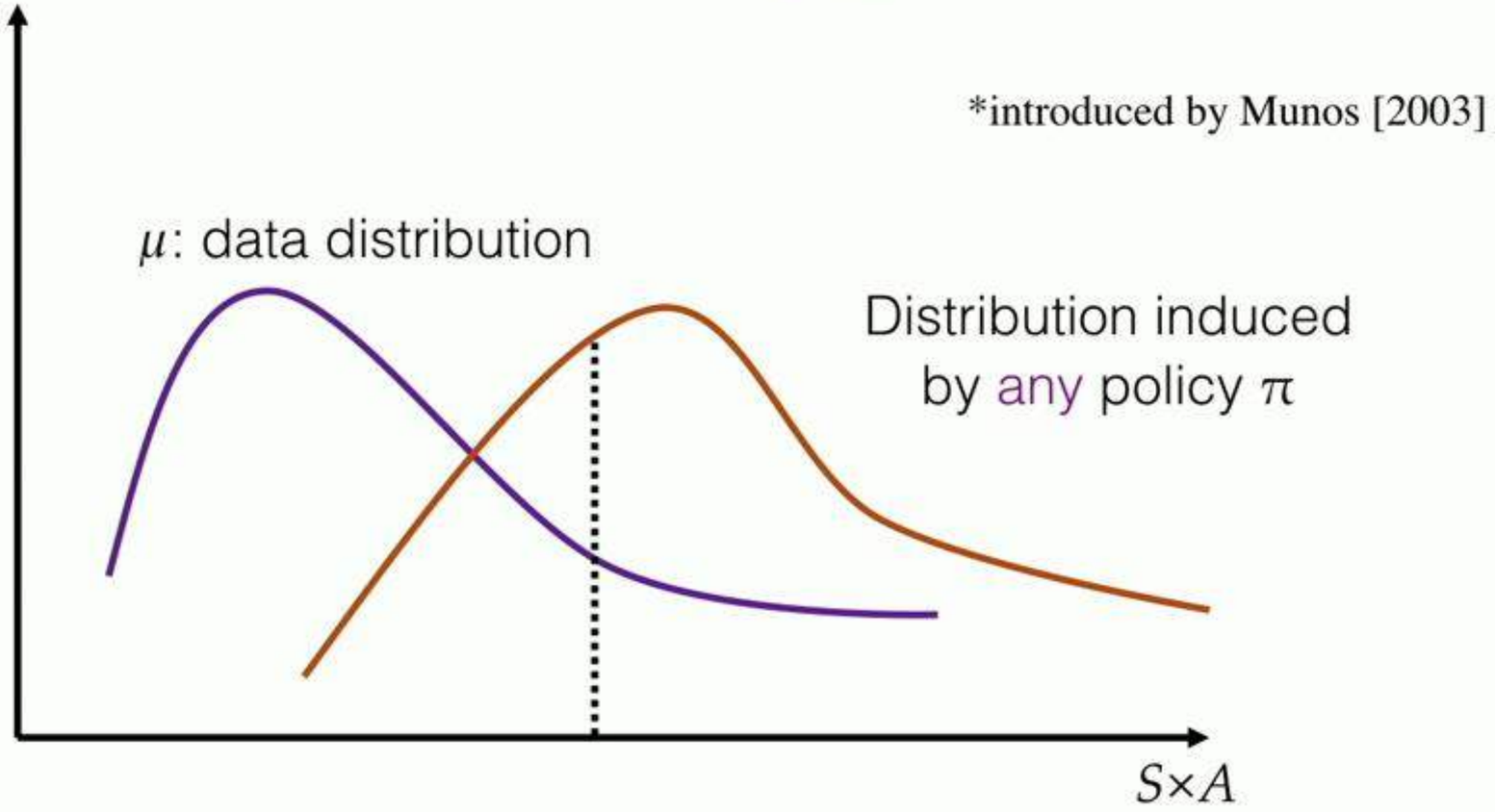

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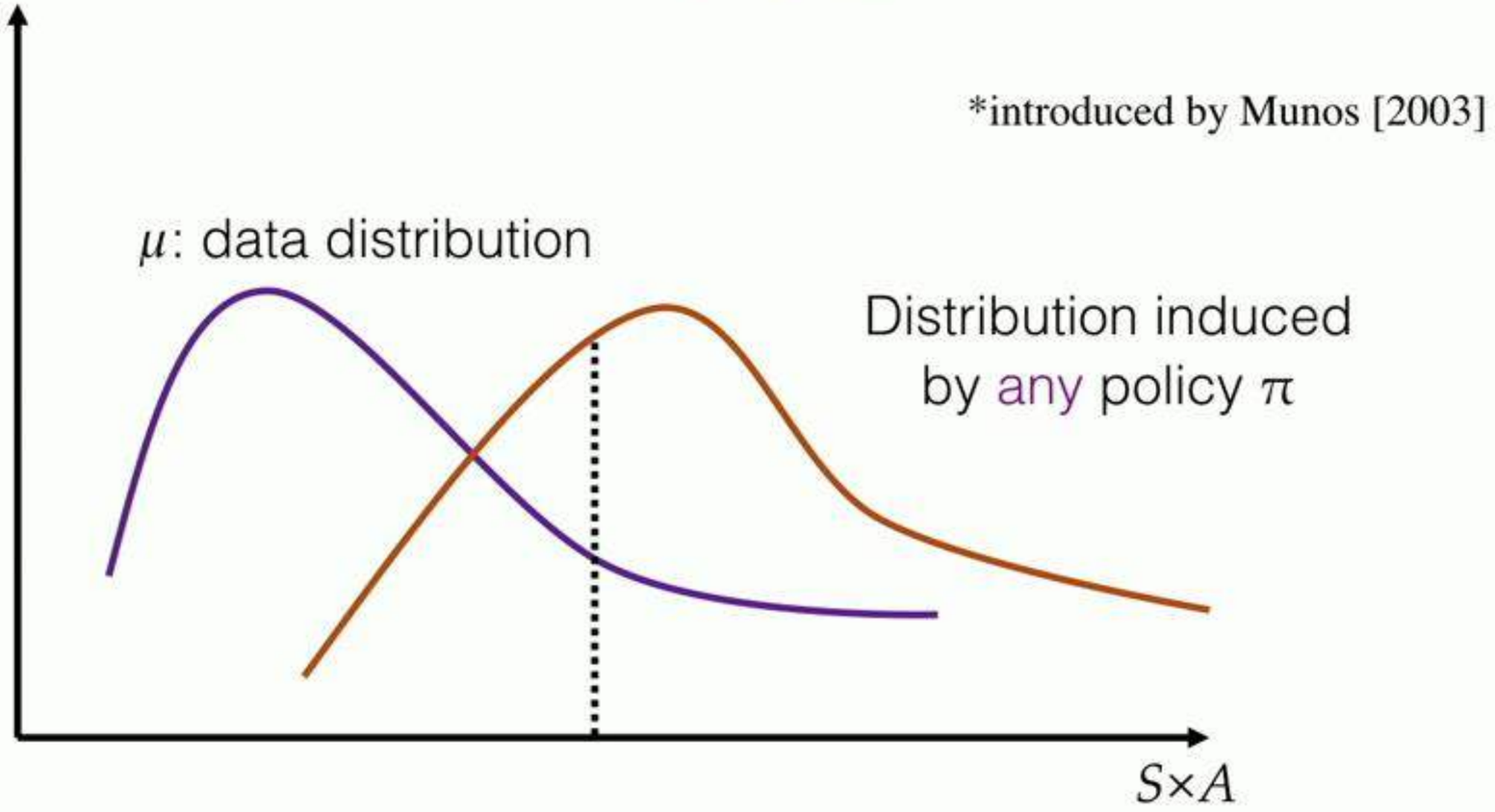

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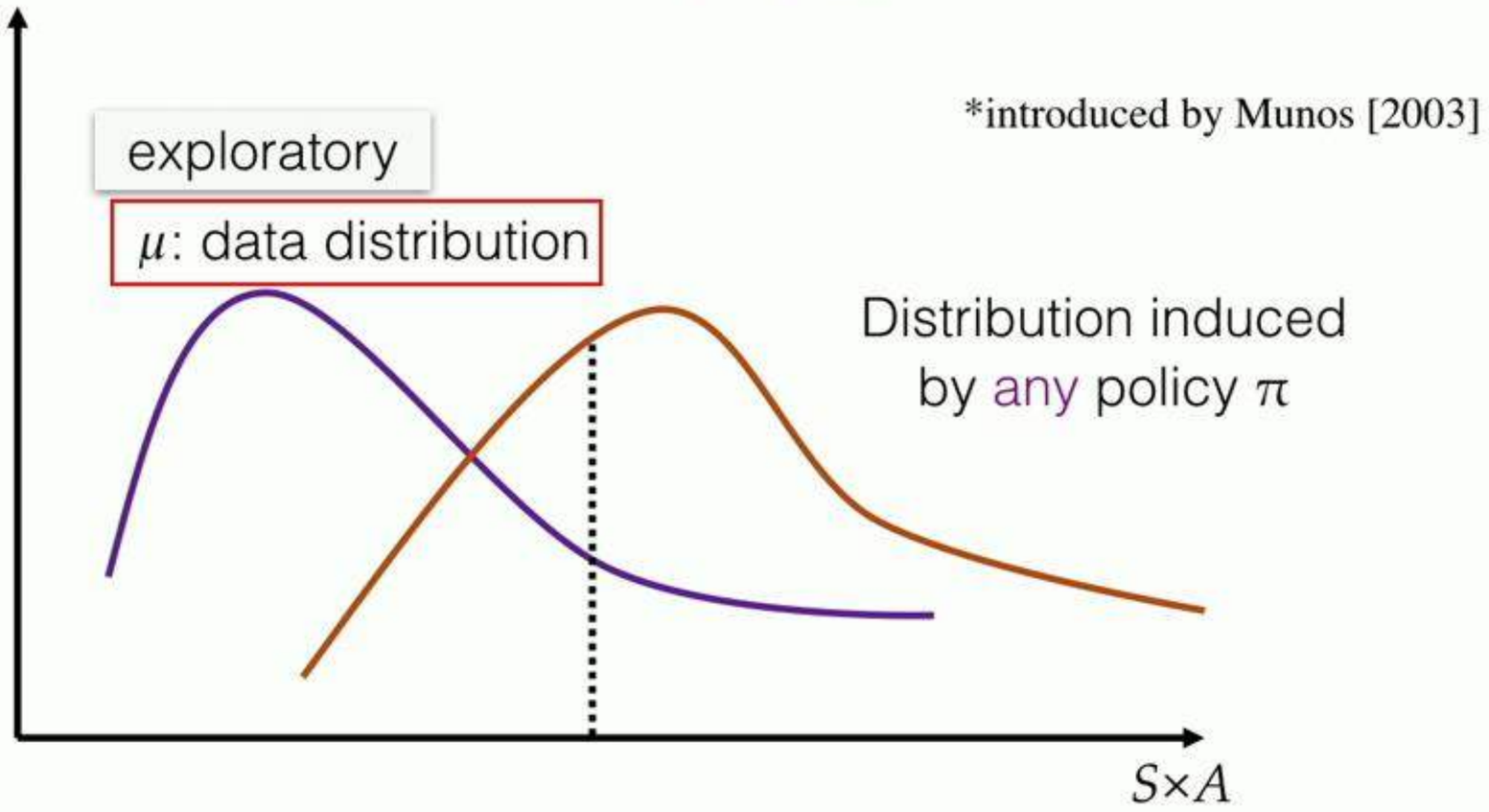

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Typical algorithm: Fitted Q-Iteration (FQI) [Ernst et al'05]

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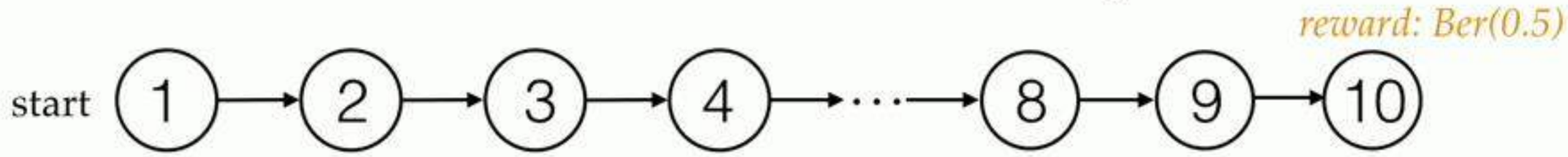
- That is, $f_t = \arg\min_{f \in \mathcal{F}} \sum_{(s,a,r,s') \in D} \left(f(s,a) - \left(r + \gamma \max_{a' \in \mathcal{A}} f_{t-1}(s', a') \right) \right)^2$

How FQI works (finite horizon, γ=1)



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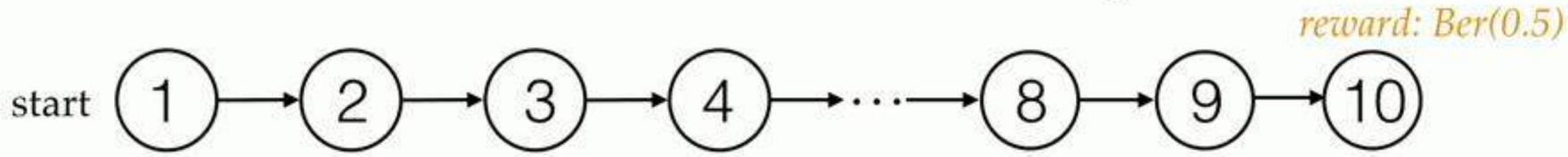

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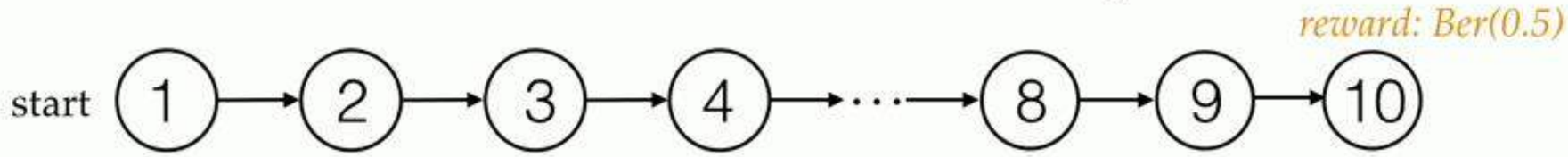

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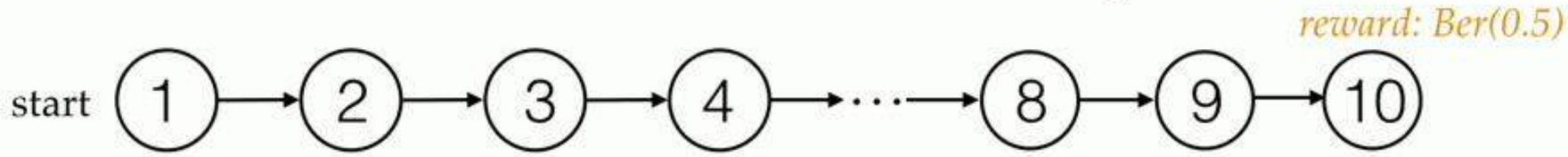

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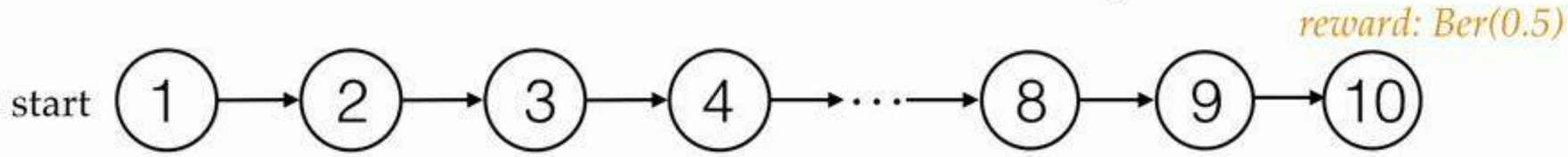

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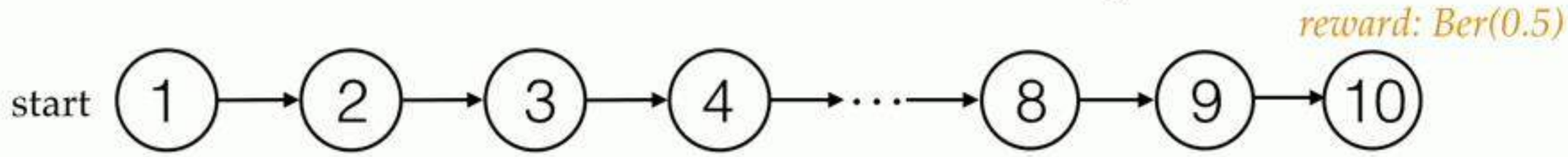

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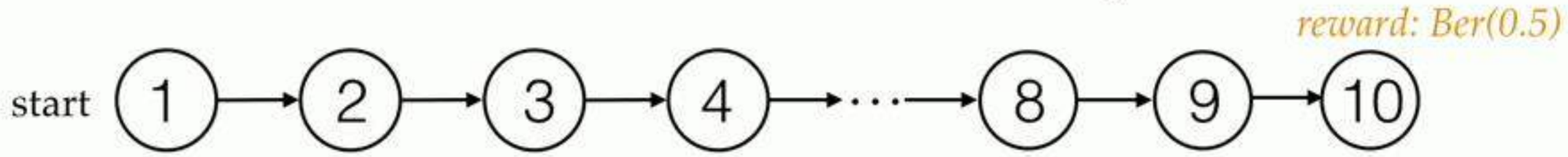

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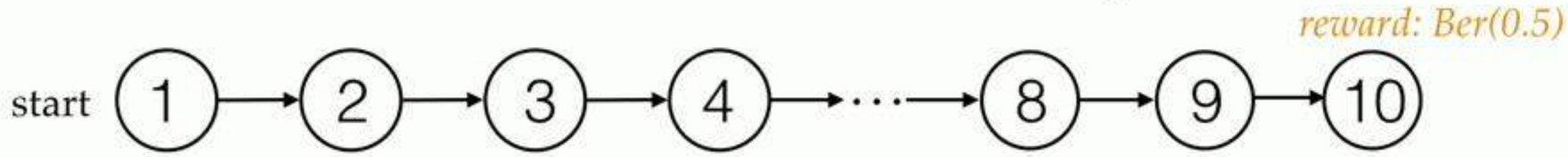

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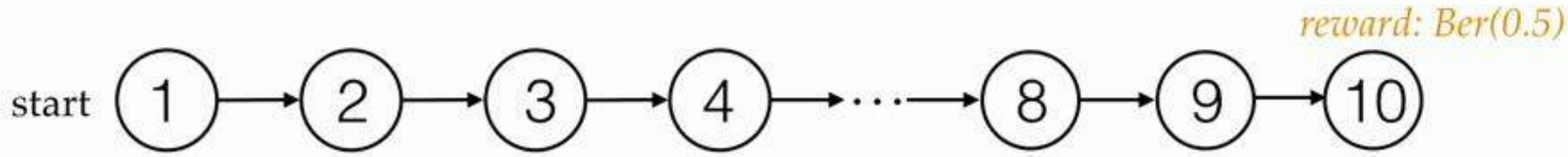

Function class

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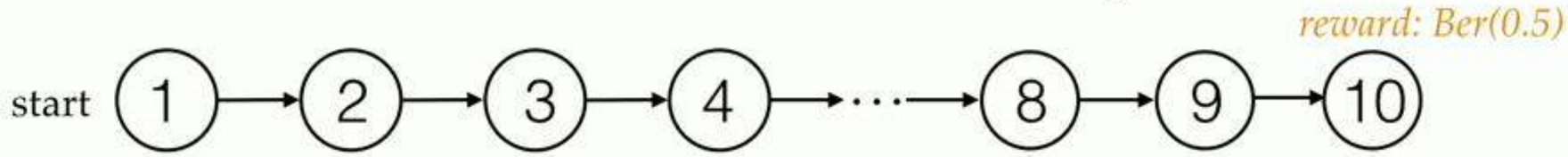

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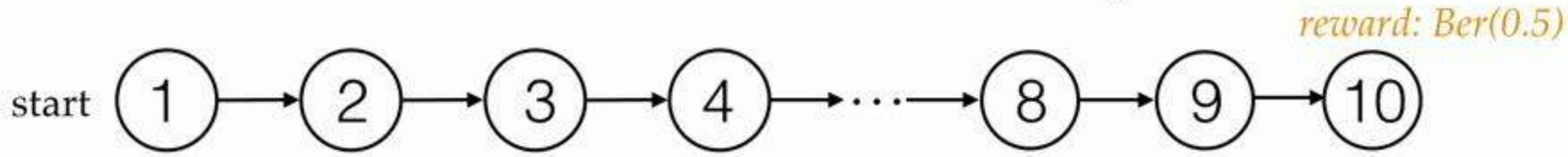

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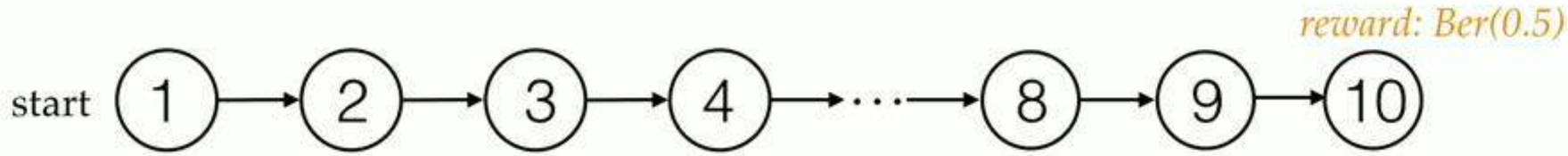

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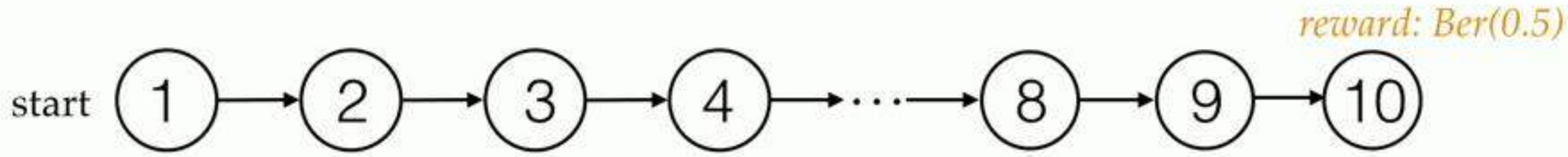
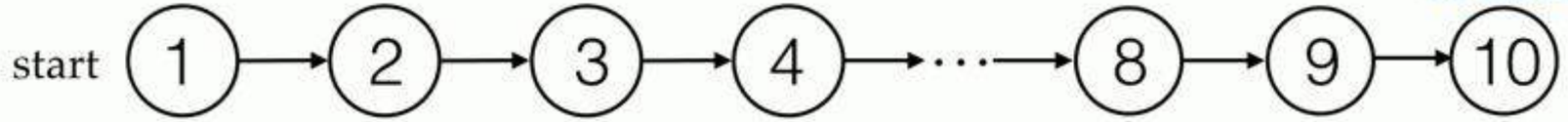


Function class	
	0.5
	0.501

How things go wrong (w/ restricted class)

reward: Ber(0.5)



Function	0.5	×	0.5	×	0.5	×	…	×	0.5	×	0.5	×	0.5
class	1.012		0.756		0.628				0.504		0.502		0.501

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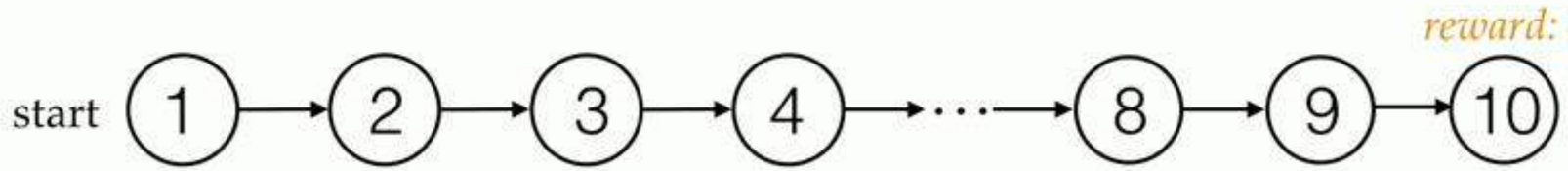

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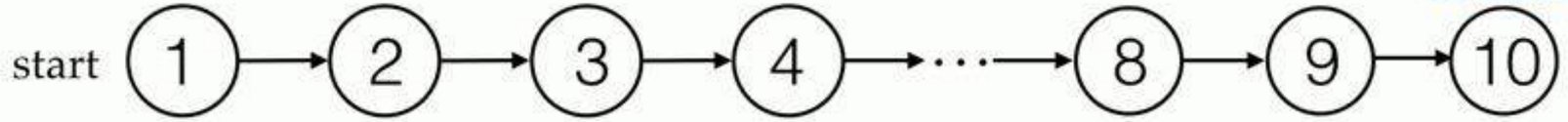

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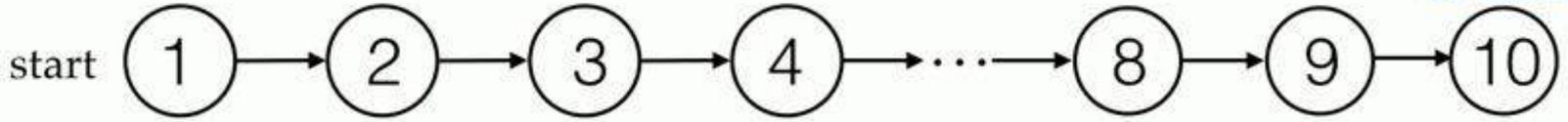

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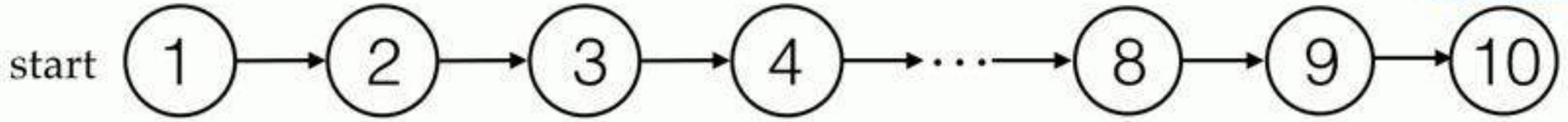

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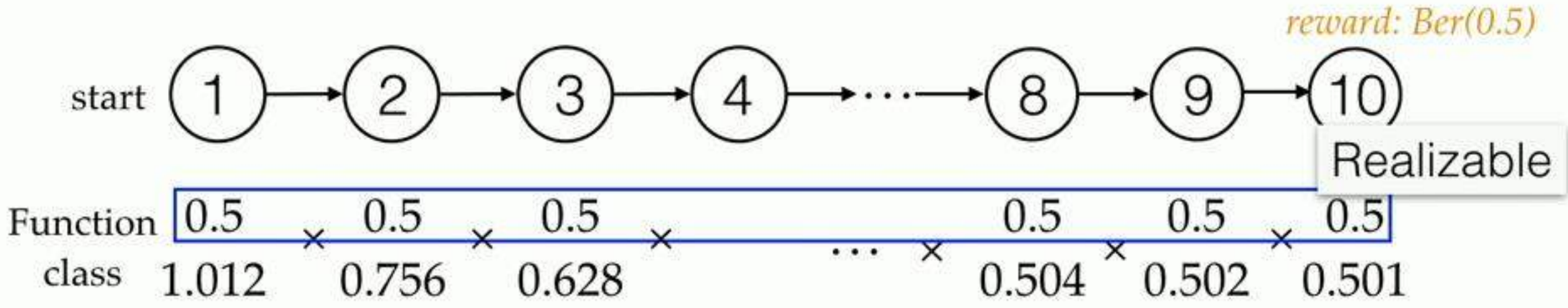

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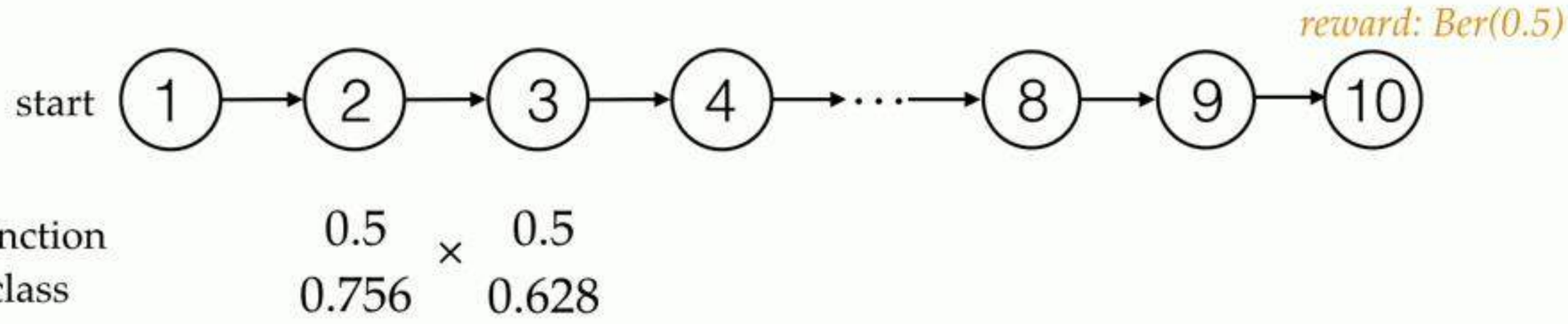
Example given in Dann et al'18

- similar conclusion known for decades (e.g., Van Roy'94)

Fix using a strong assumption ("completeness")



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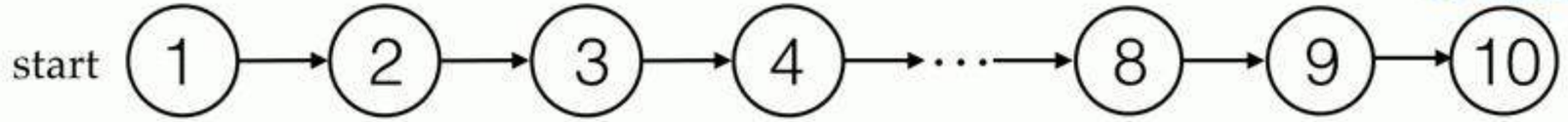

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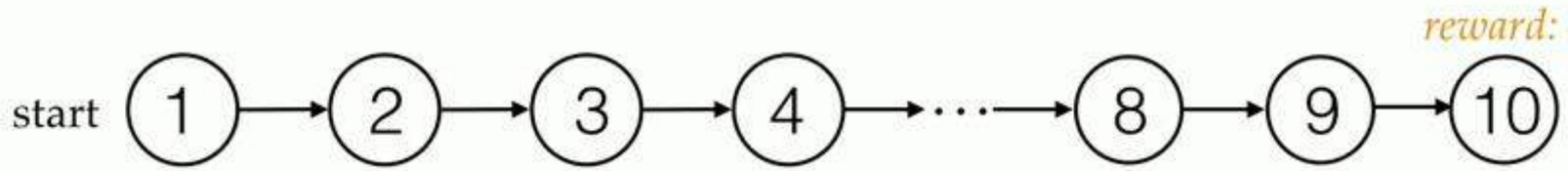

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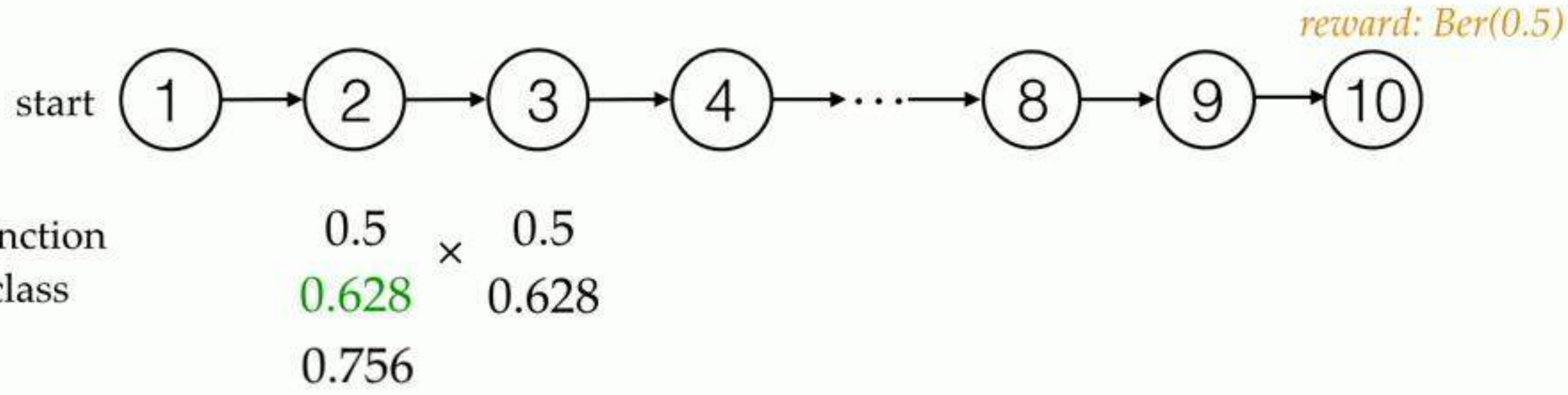

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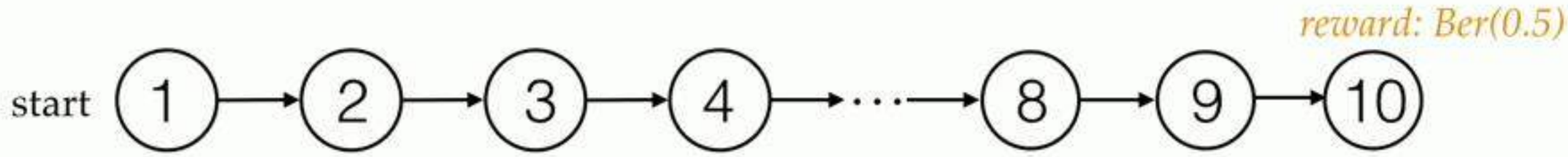

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	0.628		0.628
	0.756		

*introduced by Szepesvari & Munos [2005]

- More generally: the regression problem

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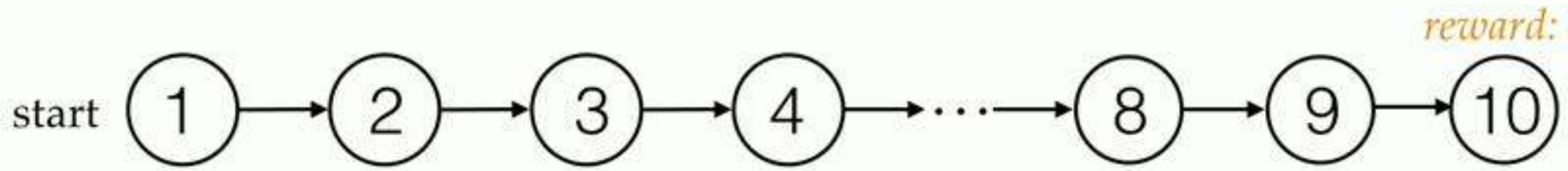

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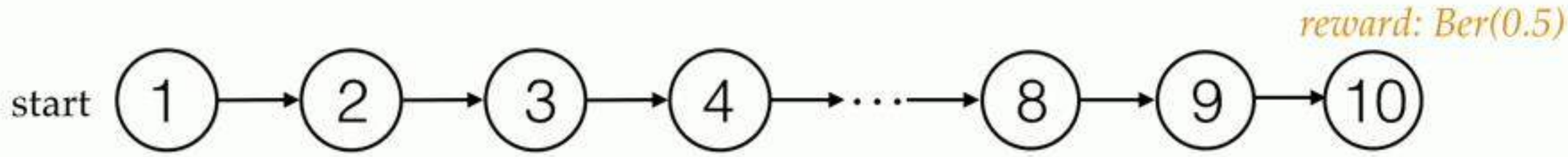

Function class

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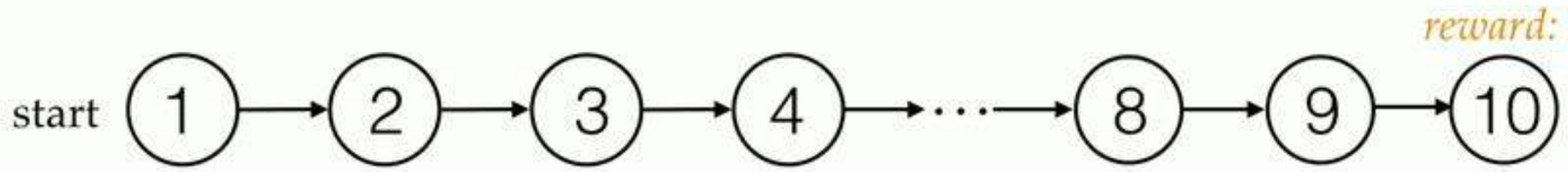

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- With this assumption, sample complexity *poly*$(\log|F|, H, 1/\varepsilon, 1/\delta, C)$

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  - Will get back to its importance at the end

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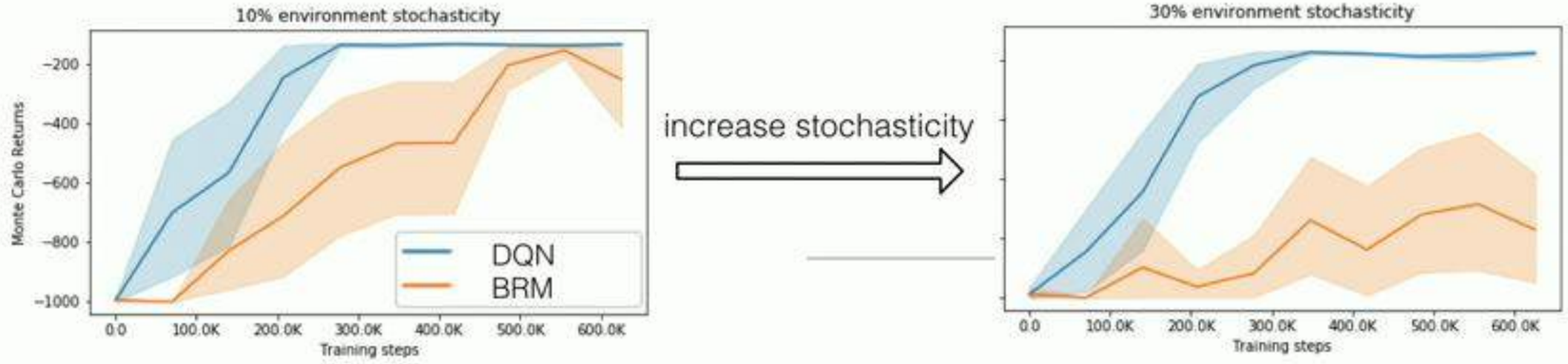

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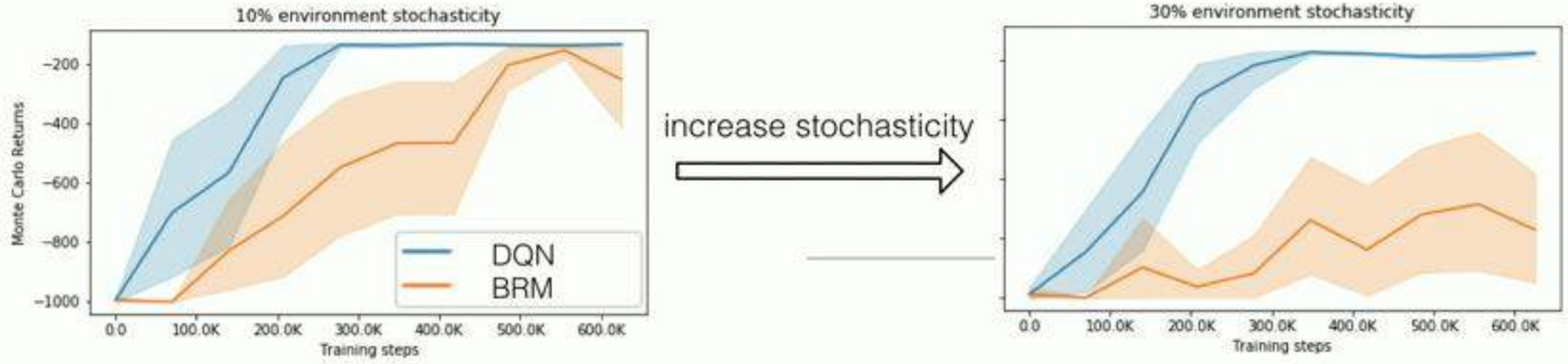

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Importance of the conjecture

RL tractable & challenging

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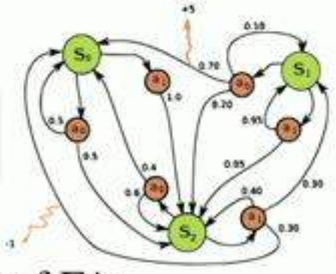

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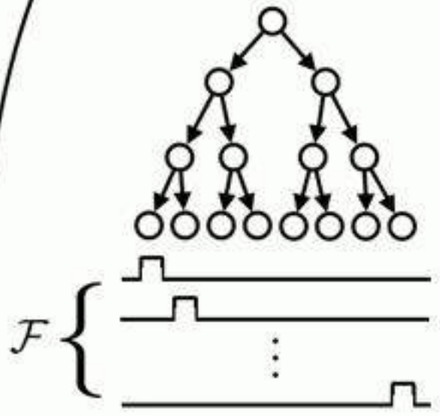

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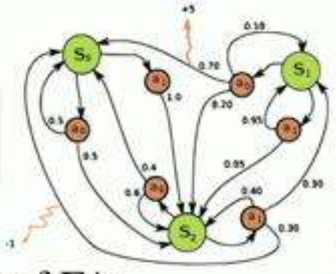

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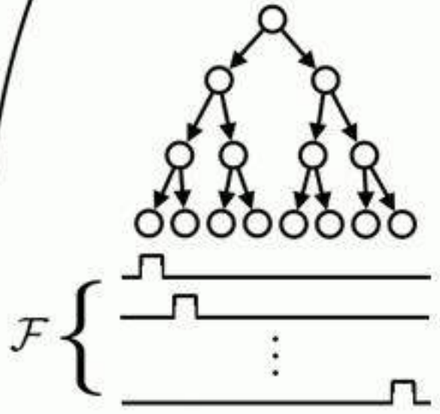

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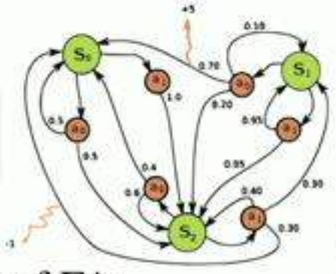

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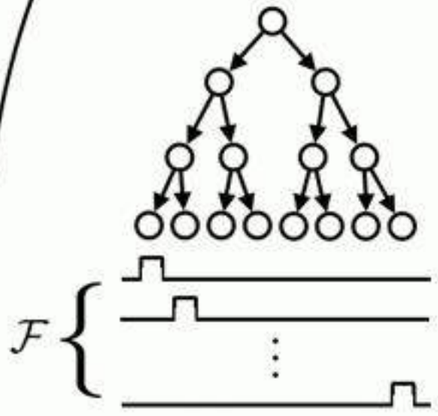

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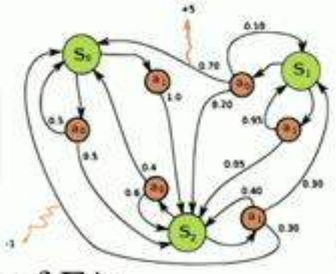

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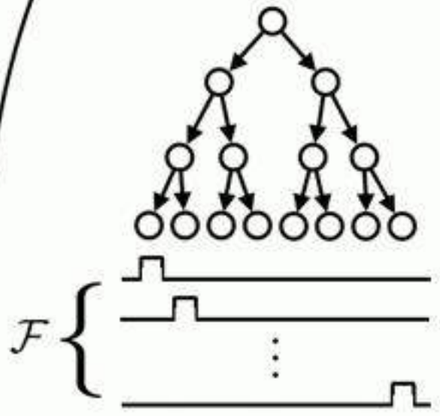

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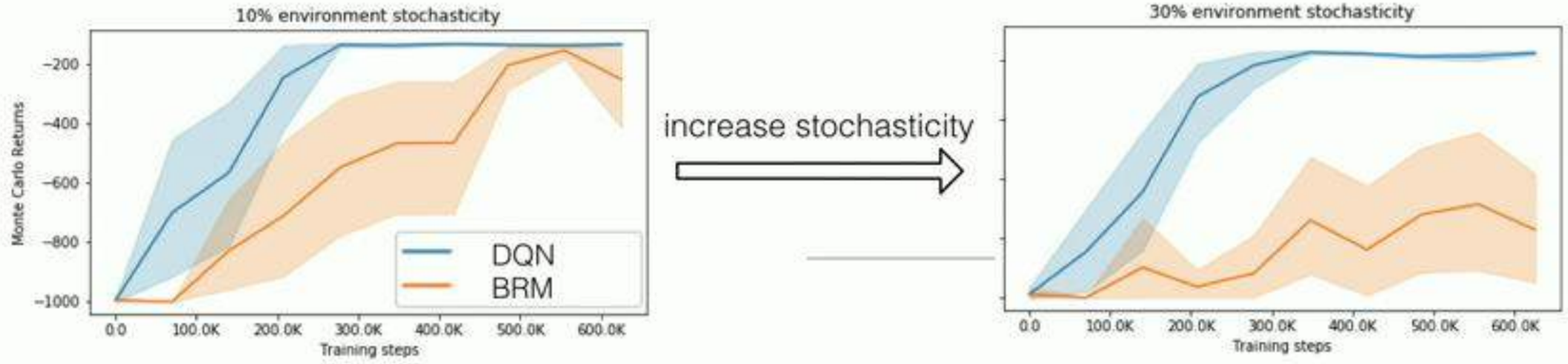

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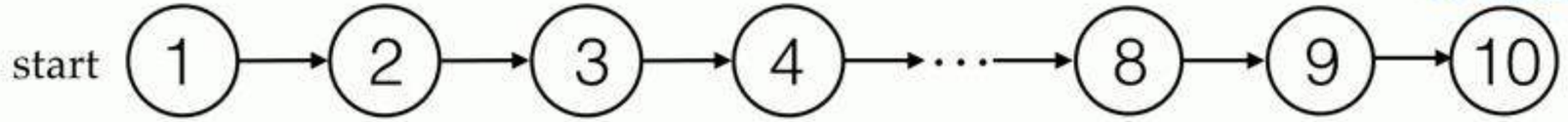
Fix using a strong assumption ("completeness")

reward: Ber(0.5)



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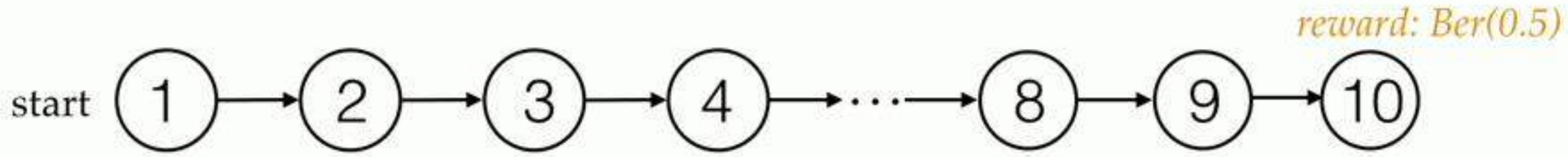
How things go wrong (w/ restricted class)



Function class	0.5	×	0.5	×	0.5	×	… ×	0.5	× 0.5 × 0.5
	1.012		0.756		0.628			0.504	0.502 0.501

FQI Iter #1: Data: (⑩, 1, *end*), …, (⑩, 0, *end*) ⇨ 0.501

Iter #2: Data: (⑨, 0, ⑩) ⇨ (⑨, 0+0.501) ⇨ 0.502 0.501

…

Iter #10: !!! 1.012 0.756 0.628 … 0.502 0.501

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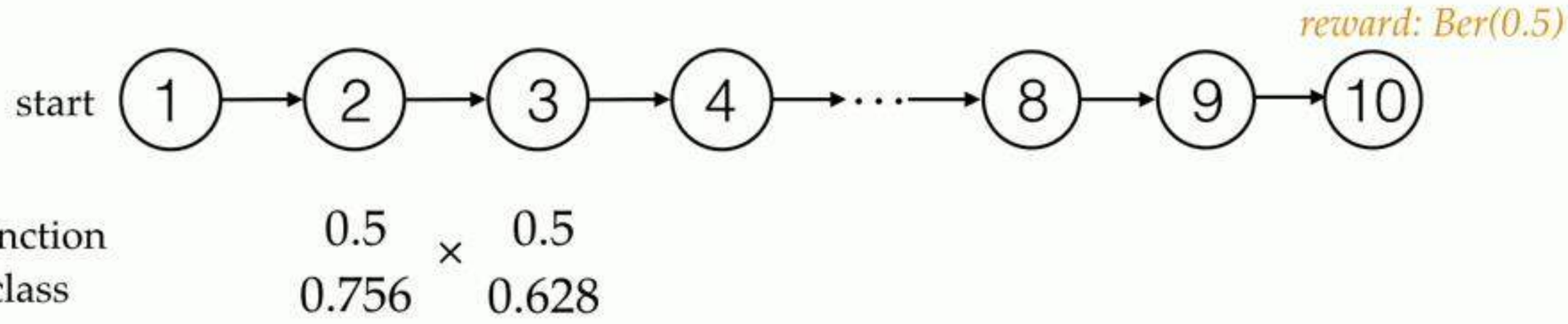

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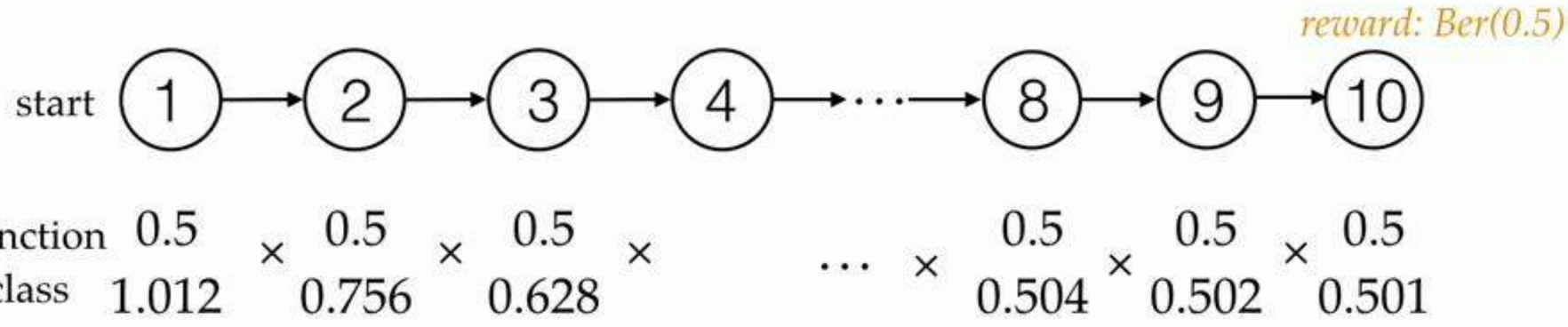

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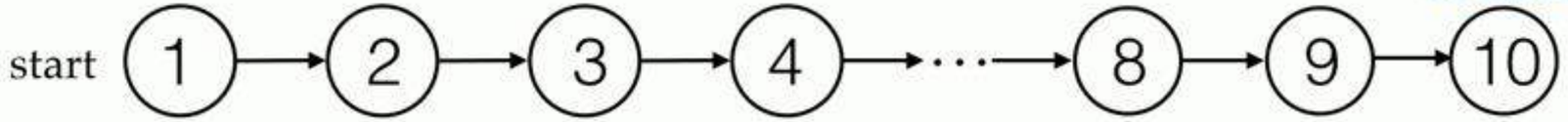

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